

COMPRESSIVE BUCKLING OF PERFORATED PLATE ELEMENTS

By W. Pennington Vann¹Rice University
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INTRODUCTION

Light-gage steel members are frequently perforated in order to provide access for plumbing, electrical wiring, and other services. Sometimes isolated holes are punched at particular locations; in other cases the member is manufactured with a regular pattern of holes. With a member of the latter type, the holes may also be used in attaching collateral material.

Figure 1 shows a few hole patterns currently in use by a major manufacturer of light-gage building components. Typically, such holes are cut in the web of a channel or built-up I section, so the perforated plate element is supported along two parallel edges. Geometric parameters which might affect the behavior include the number of holes, their shapes, their sizes relative to the overall dimensions of the plate, and their positions with respect to one another.

Another important factor affecting the performance of a perforated plate is the type of applied stress. Figure 2 illustrates four basic, idealized loading conditions which might be considered -- pure tension, pure compression, pure shear, and pure bending. Realistic situations usually involve combinations of these conditions. The two most important examples are beams in bending and shear (a combination of cases (c) and (d)), and columns or studs in pure compression (case (b)).

¹Assistant Professor of Civil Engineering, Rice University, Houston, Texas

This paper is concerned with the strength of a light-gage steel member in axial compression when one or more of the plate elements of which the member is composed has circular holes along its centerline. First, sample analytical results are given for the elastic buckling of a plate with a single hole. This information is intended to provide some insight into the effect of the hole and the expected influence of a stiffening lip around the edge of the hole. Next, some experimental results are given for the elastic buckling of analogous light-gage elements. Finally, results are presented for the ultimate strength of several light-gage wall studs which are perforated all along their length.

ELASTIC BUCKLING - TYPICAL ANALYTICAL RESULTS

An appreciation of the effect that a central circular hole has on the elastic buckling of a flat plate may be gained by analyzing a square simply supported plate. Solutions have been presented for this case by Levy, Woolley, and Kroll,⁽²⁾ by Schlack,^(3,4) and by Kawai and Ohtsubo.⁽¹⁾ The results to be presented here have been obtained by a finite element formulation,^(7,8) and are part of a more extensive investigation of the behavior of perforated plates than has been carried out to date.

Unreinforced Holes. Consider first the thin plate shown in plan in Fig. 3. The plate is square, with a side dimension B , and it has a central circular hole of diameter D . The boundary conditions for bending (that is, for out-of-plane deflections) are simple supports along all four edges. The plate is compressed in one direction by imposing uniform in-plane displacements of magnitude $eB/2$ along the top and bottom edges. Thus the average in-plane strain in the direction of the loading is e . The unloaded edges are free to move in or out and to deform in the plane of the plate. These boundary conditions are reasonable for the web of a channel or I-section.

Figure 4 shows a typical finite element mesh used to model the perforated plate.

The circular hole must be represented by a polygon, and the area of the polygon is set equal to the area of the hole.

For a proper understanding of the buckling behavior of the pierced plate, it is useful to separate the effect of the hole upon the in-plane stress distribution from the effect of the hole upon the bending stiffness of the plate. Naturally, one would expect the hole to weaken the plate and to reduce the buckling load, and this is true with regard to the second effect listed. The hole does reduce the bending stiffness in the central portion of the plate, and this does tend to lower the buckling load. However, the effect of the hole on the in-plane stress distribution tends to increase the buckling load. This is because the hole causes the in-plane stresses to flow away from the central portion of the plate, which is the part that is most prone to buckle, and toward the sides, which are partially restrained against out-of-plane deflection.

Figure 5 shows how the two cited effects interact as the hole sizes increases from zero to one-half the width of the plate. The heavy solid line with open circles gives the average edge stress at buckling, normalized with respect to the theoretical value for no hole, and the light solid line with open triangles gives the corresponding ratio for the nominal strain at buckling. From the standpoint of strength, the average edge stress is of greater interest because it is a measure of the total in-plane load. Note that when the plate has a hole, the in-plane stresses are not uniformly distributed along any horizontal section. The edge stresses tend to be lower in the center than at the sides, and the peak stresses at the sides of the hole are generally several times the average edge stress.

Figure 5 shows that as the hole sizes increases from zero, the buckling load at first decreases, making it appear that the loss of bending stiffness dominates, but then the load increases again and comes back up to the value for no hole at $D/B = 0.5$.

For still larger holes the load continues to increase, indicating that the dominant factor is the flow of in-plane stresses to the sides. From a practical standpoint, the most important conclusion from this figure is that, for the particular boundary conditions considered, the total buckling load never falls below 90% of the value for no hole.

A more intuitive appreciation of this behavior may be gained from the following simplified analysis, which was first suggested by Kawai and Ohtsubo.⁽¹⁾ Since the hole tends to relieve the compressive stress in the central portion of the plate, consider the entire load to be carried in uniform stress by the two rectangular strips left on each side of the hole when the material above and below the hole is removed. These strips are shown shaded in the inset in Fig. 5. If each strip is considered to be simply supported on three sides and free on the side next to the hole, then known solutions⁽⁶⁾ provide the results shown by the dashed lines in Fig. 5. For small holes this analysis is overly conservative because the bending stiffness is changed more than the in-plane stress distribution. However, the results are reasonably accurate for larger hole sizes.

It might be noted before leaving Fig. 5 that the critical strain never is less for the pierced plate than for the corresponding unpierced plate, and the ratio between the two values becomes quite large for large hole sizes. This is because the overall in-plane stiffness of the plate is always reduced by the hole more than the buckling stress may be reduced.

Holes With Stiffening Lips. Like the hole itself, a stiffening lip around the edge of a hole has two opposing effects on the buckling load of the plate. To the extent that the reinforcement provides in-plane stiffening, it attracts the in-plane stresses back to the center of the plate, and this tends to lower the buckling load. On the other hand, to the extent that the reinforcement resists out-of-plane rotation at the edge of the hole (both along the edge and normal to the edge), the overall stiffness of the plate against

out-of-plane deflection is increased, and thus the buckling load is increased.

In order to estimate the relative influence of these two effects, elastic buckling results are presented in Table A for the stiffened plate shown in Fig. 6. In this example, the diameter of the hole is 0.3 times the plate width, the thickness of the stiffener is the same as that of the plate, and the height of the stiffener is ten times its thickness. Although a lip formed around a hole in a light-gage section would normally project only to one side of the plate, the stiffener in Fig. 6 is concentric about the middle surface of the plate. The concentric stiffener is considered because, in principle, a plate with an eccentric stiffener would not buckle. Instead, it would deflect out of its original plane upon initial in-plane loading. Nevertheless, the behavior of such a plate could be better understood if the buckling load were determined for the equivalent concentrically stiffened plate, that is, a plate having a concentric stiffener with the same cross-sectional area, torsional rigidity, and moment of inertia about the middle surface of the plate.

The buckling quantity, \bar{s}_{cr} , shown in Table A is the average edge stress at buckling normalized with respect to Young's modulus, E, and the width-to-thickness ratio, B/T:

$$\bar{s}_{cr} = \frac{s_{cr}}{E \left(\frac{T}{B} \right)^2} \quad (1)$$

where s is the average edge stress (the total in-plane load divided by the area of the edge). The first line in Table A gives the finite element buckling value for no hole, which is 2.9% above the exact value of $\pi^2/3(1 - \nu^2) = 3.615$. The computed value was obtained using the mesh shown in Fig. 4, but with the hole filled in by three additional elements meeting at the center of the plate. The values in the second and third lines of Table A were found with the unaltered mesh of Fig. 4. The second line shows that \bar{s}_{cr}

for the hole with no stiffener is 8% less than that for no hole. The third line shows that the stiffener causes the buckling load to be 54% greater than for the unstiffened hole, and 41% larger than for the unpierced plate.

From these results it may be concluded that, with regard to elastic buckling, the benefit from the bending stiffness of the lip far outweighs the adverse effect of the in-plane stiffening. Since the total volume of the stiffener is only 33% greater than the volume of the material removed in cutting the hole, the in-plane stress distribution would not be expected to differ much from that of the unpierced plate. On the other hand, the material far from the middle surface of the plate is very effective in resisting bending along the direction of the stiffener.

ELASTIC BUCKLING -- EXPERIMENTAL RESULTS

Figure 7 shows the specimen and the loading scheme used in the experimental study.⁽⁵⁾ One 14-gage steel channel section with a nominal 8-inch web and lipped flanges was used for all of the tests. The actual dimensions are shown in the figure. The width-to-thickness ratio of the web was 100. Four circular holes of successively larger diameter were cut in the web in the same location, and after each hole was cut several buckling tests were conducted. Also, before each larger hole was cut, a half-inch width of the plate around the edge of the previous smaller hole was bent out at approximately 45° to form a stiffening lip in the shape of a cone, and the buckling load for the resulting hole was determined. The nominal hole diameters were 1", 2", 3" and 4".

The specimen was bent about the weak axis with the web on the compression side. Two symmetrically placed loads were used, so that the central segment of the web was placed in uniform compression. This central segment was approximately five times the flat width of the web. Reinforcing bars were welded to the flanges of the section to prevent

them from yielding prematurely in tension.

One purpose of the testing program was to evaluate different experimental criteria for the buckling of irregular plates. The average buckling stresses to be presented herein were determined by a modification of the Southwell concept, using data from strain gages placed in pairs on the top and bottom surfaces of the web. The locations of the gages are shown by the dark rectangles in the upper part of Fig. 7.

Figure 8 shows a picture of the specimen with a one-inch diameter unflanged hole, and Fig. 9 shows how the specimen was loaded in a standard universal testing machine. The lower (support) spreader beam was rested directly on the lower head of the machine, and the upper (loading) spreader beam was hung from the middle head by means of a universal joint. Figure 10 shows a detail of the load and support points, as well as the clip angles that were used to keep the flanges of the specimen from spreading as the load was increased. This was necessary in order to keep an associated dishing of the web from affecting the results. Of course, the presence of the clip angles also meant that the lateral edges of the web were not simply supported, but instead were elastically restrained against rotation.

In Fig. 11 are shown the experimental results obtained for the average stress far from the hole at elastic buckling. The data are given in ksi, so that the buckling values may be compared to possible yield stress values. For the larger holes the peak stresses next to the hole were much larger than the average stresses shown in the figure. The square data points are for unstiffened holes (including the case of no hole), and the triangular points are for stiffened holes.

The test results are all greater than the corresponding finite element results presented earlier for square plates and simple supports. These finite element results are shown in Fig. 11 by the solid line. The differences are attributed to the rotary

restraints along the lateral edges of the test plate, and to the fact that the test plate was oblong, rather than square. The latter difference affects the in-plane stress distribution when the plate is pierced. For the unpierced plate the proximity of the experimental buckling value to the theoretical value for fixed lateral supports, which is shown along the left margin, indicates that the rotary restraints were almost rigid. However, it is expected that the experimental values in the figure are slightly high.⁽⁵⁾

In any case, the buckling stresses for the unreinforced holes do follow the trends predicted by the theory, for they vary by only a few percent within the range of hole sizes considered. Also, the reinforcing lips always increase the buckling load by 50% or more over the value for an unstiffened hole of the same size. This effect was predicted by the analytical data as well. The stiffening lips used in the tests are well represented by the lip considered in the analysis.

EXPECTED ULTIMATE STRENGTH

The ultimate strength of a thin plate loaded in compression is not limited by elastic buckling. However, if certain assumptions prove to be valid in the postbuckling range, then the foregoing results may be used to estimate the ultimate strength of the perforated plate. The plate may then be designed in a rational way, that is, a design stress based on the ultimate strength may be specified for a given perforated element, or a hole size may be chosen for a given flat width and thickness.

The following assumptions appear to be reasonable:

- a. cutting a relatively small hole in a plate will change the plate's post-buckling behavior no more than it changes the plate's buckling behavior, which appears to be less than about 10%;
- b. the effective width concept may still be used in the postbuckling range;

and c. first yielding, which may be expected to occur next to the hole rather than at the outer edge of the plate, will not have much effect on the strength.

With regard to the third assumption, it would seem that if the buckling load is not greatly reduced by cutting out more of the plate entirely, then some yielding at the edge of the hole prior to yielding elsewhere should not be very detrimental. However, all of these concepts need to be verified by theory and experiment.

If the above assumptions are correct, then the ultimate strength of a thin plate is not lowered by cutting a small unreinforced hole in its center, so long as the diameter of the hole is smaller than the width that is neglected in applying the effective width concept. With or without the hole, the ultimate strength of such a plate is governed by the effective width and the yield stress. Also, the foregoing buckling data indicate that a stiffening lip around a hole may be able to make the entire net section through the hole effective up to full yielding, including the part that might be neglected if the effective width concept were applied. This second idea would seem to be more open to question than the first, because the stiffener itself may lose its effectiveness when it begins to yield.

These two tentative conclusions are partially confirmed by the results of several recent tests in which short perforated wall studs were loaded up to failure in compression. The mineature panels tested are shown in Fig. 12. The two studs in each panel were seated in a light-gage channel, shown at the near end in the figure, and were cut off square at the other end. One panel had unpierced studs, a second panel had studs with unflanged holes $1 \frac{1}{8}$ inches in diameter spaced at $2 \frac{1}{4}$ inches, and the third panel had studs with $6 \frac{7}{8}$ -inch by $1 \frac{1}{8}$ -inch oblong holes which were reinforced all around by half-inch lips.

The length of panel was chosen as 48 inches, so that, with sheet rock attached to the sides, the studs would be expected to fail in local buckling rather than overall buckling. The B/T ratio of the web of each stud was 57.3, and the material was mild steel with a yield stress of 37.6 ksi. Considering the full width of the web to be effective, and neglecting cold working in the bends, the theoretical ultimate load would be 38.6 kips. However, considering only the effective width of the web, the theoretical ultimate load would be only 32.2 kips.

The holes in the pierced studs covered only 35% of the flat width of the web, or less than the 40% that would be neglected in an effective width calculation. Therefore, according to the concepts discussed above, the studs with unflanged holes might be expected to carry as much load as the unpierced studs, and the studs with flanged holes might be expected to carry more load than the unpierced studs.

The ultimate loads obtained in the tests are shown in Table B, where P_u is the ultimate load on a single stud. The values are all within 7% of the theoretical value for no web holes. These data tend confirm the expectation that small unflanged holes have little effect, but they fail to show the expected influence of stiffening lips. Further tests are needed to clarify these results.

CONCLUSIONS

The theoretical and experimental results presented indicate that unless a central unflanged hole is fairly large, it will have a very small effect on the elastic buckling load of a plate, and that a flanged hole can be expected to make the elastic buckling load greater than that of the corresponding unpierced plate. Tentative results concerning the ultimate strength of pierced plates indicate that a small unflanged hole has essentially no effect on the ultimate strength, and a stiffened hole may or may not affect the ultimate

strength.

ACKNOWLEDGEMENTS

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APPENDIX -- NOTATION

B = plate width

D = diameter of central circular hole

E = modulus of elasticity

P_u = ultimate axial load on a wall stud

e = average in-plane strain

s = average edge stress

\bar{s}_{cr} = non-dimensional average stress at buckling, see Eq. (1)

ν = Poisson's ratio

TABLE A
EFFECT OF HOLE AND STIFFENER ON AVERAGE BUCKLING STRESS
(Simply Supported Square Plate; Circular Hole with D/B = 0.3)

Case No.	Description	Critical Avg. Stress, \bar{s}_{cr}	Ratio to \bar{s}_{cr} for no hole (Case No. 1)	Ratio to \bar{s}_{cr} for unstiffened hole (Case No. 2)
1	Plate with no hole	3.72	1.00	1.09
2	Plate with unstiffened hole	3.42	0.92	1.00
3	Plate with stiffened hole (see Fig. 6)	5.25	1.41	1.54

TABLE B
MEASURED ULTIMATE LOADS ON WALL STUDS
(B/T = 57.3; Yield Stress = 37.3; D/B = 0.33)

Case No.	Description	Ultimate Load, P_u , ksi.	Ratio to P_u for no hole (Case No. 1)	Ratio to P_u for unstiffened hole (Case No. 2)
1	Stud with no hole	29.9	1.00	0.94
2	Stud with unstiffened hole	31.7	1.06	1.00
3	Stud with stiffened hole	31.5	1.05	0.99

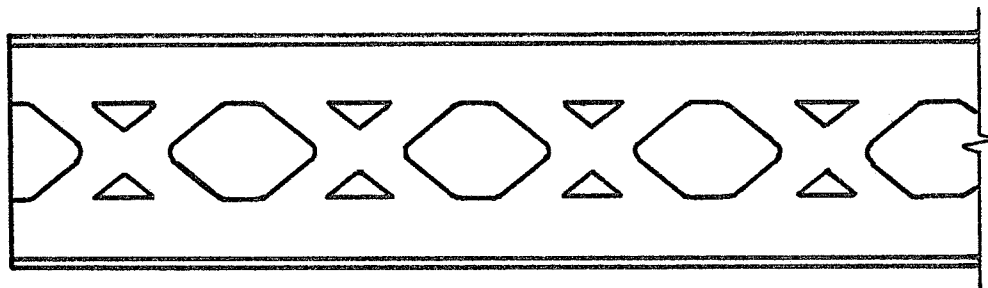
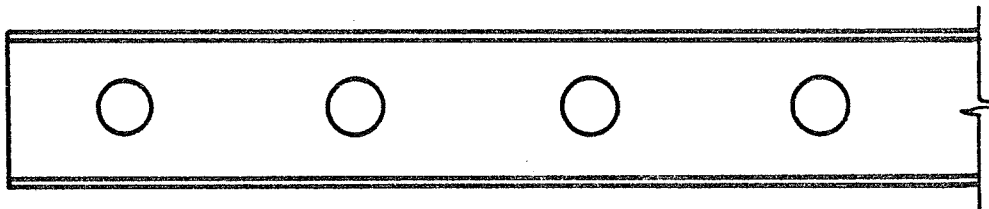
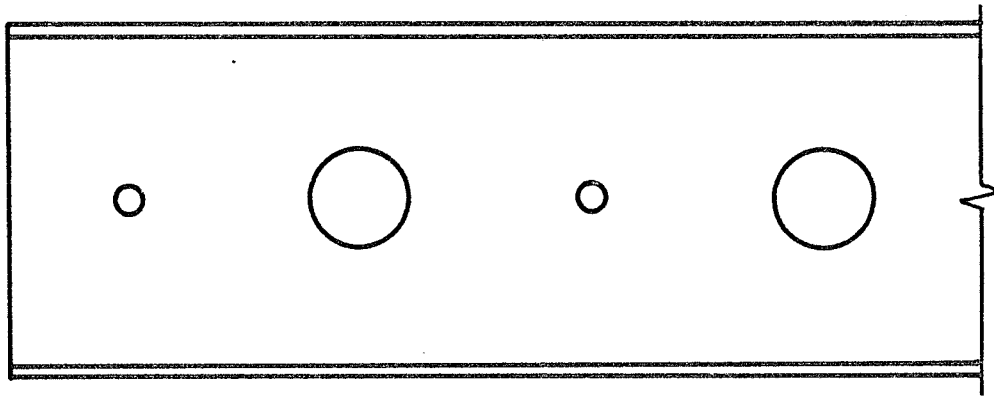
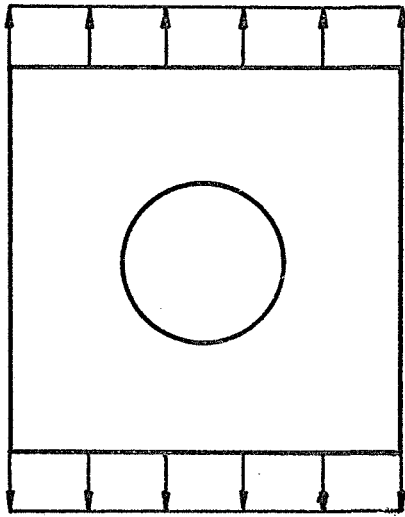
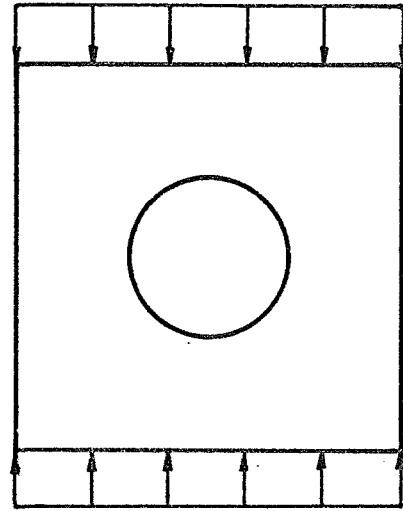


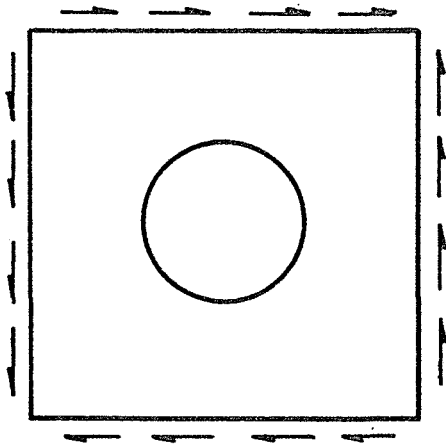
FIG. 1. EXAMPLES OF PERFORATED WEBS OF LIGHT-GAGE METAL MEMBERS



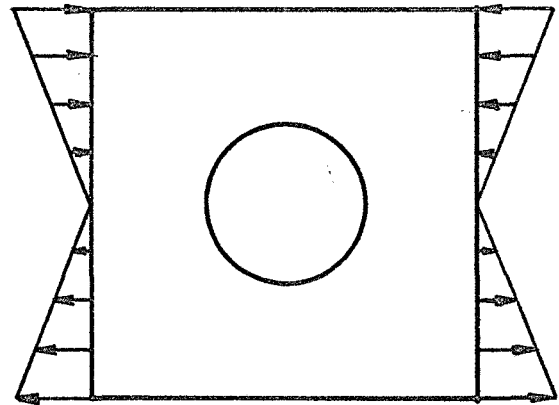
(A) PURE TENSION



(B) PURE COMPRESSION



(C) PURE SHEAR



(D) PURE BENDING

FIG; 2. TYPES OF STRESS FOR PLATE ELEMENTS

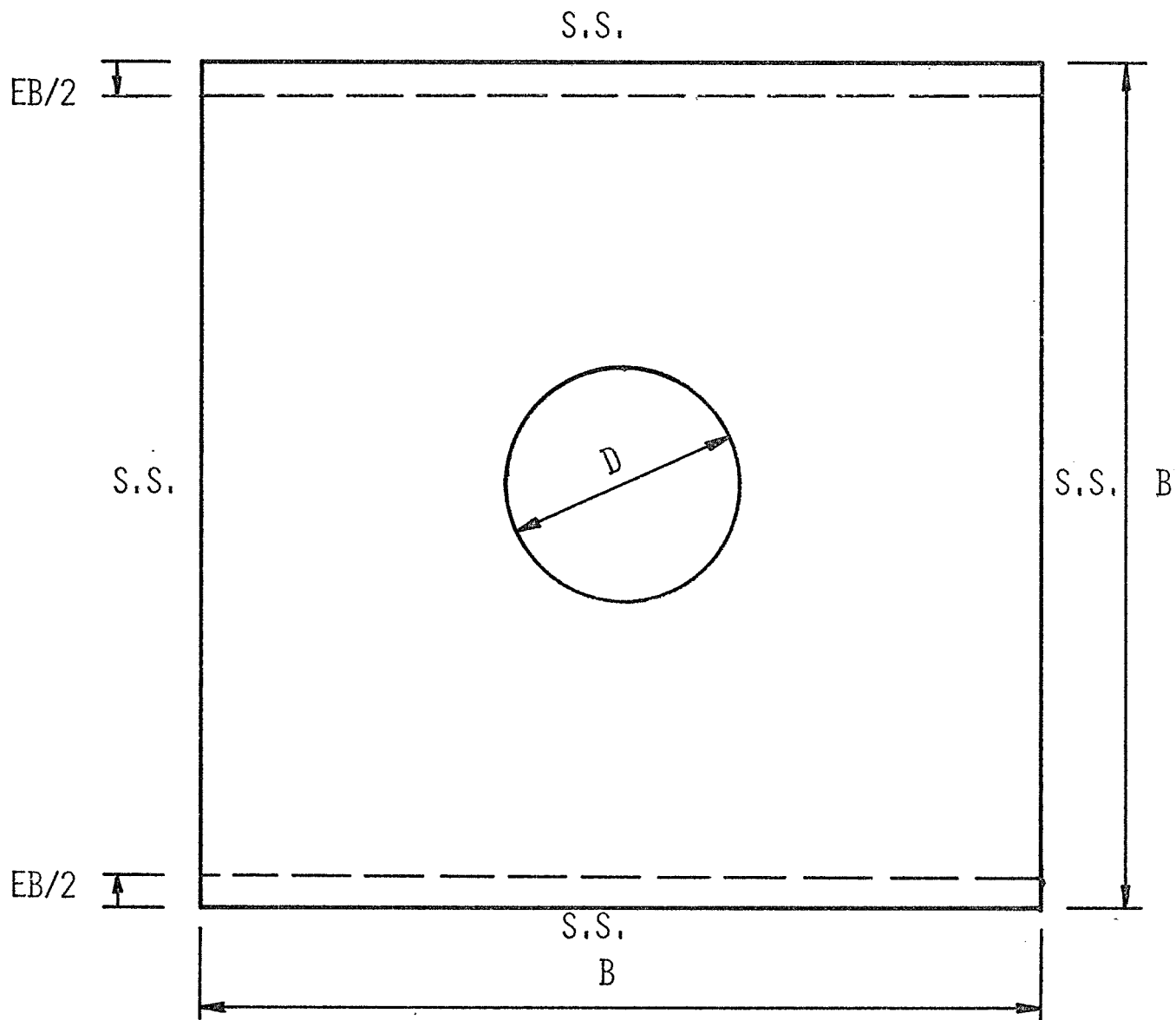


FIG. 3. BASIC CASE CONSIDERED -- SQUARE SIMPLY SUPPORTED PLATE WITH CENTRAL CIRCULAR HOLE SUBJECTED TO UNIFORM EDGE DISPLACEMENT

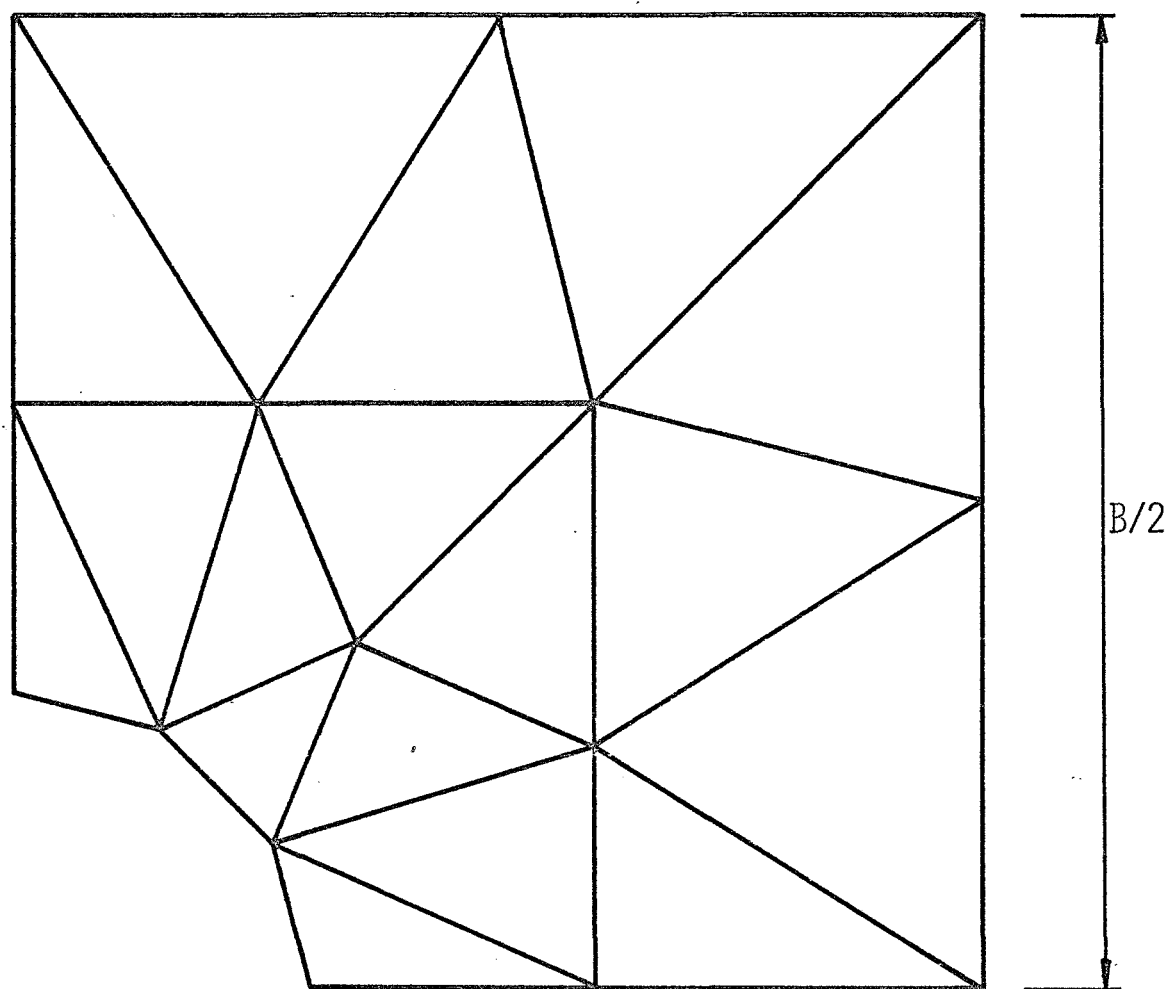


FIG. 4. FINITE ELEMENT MESH USED FOR $D/B = 0.3$
(ONE-QUARTER OF PLATE)

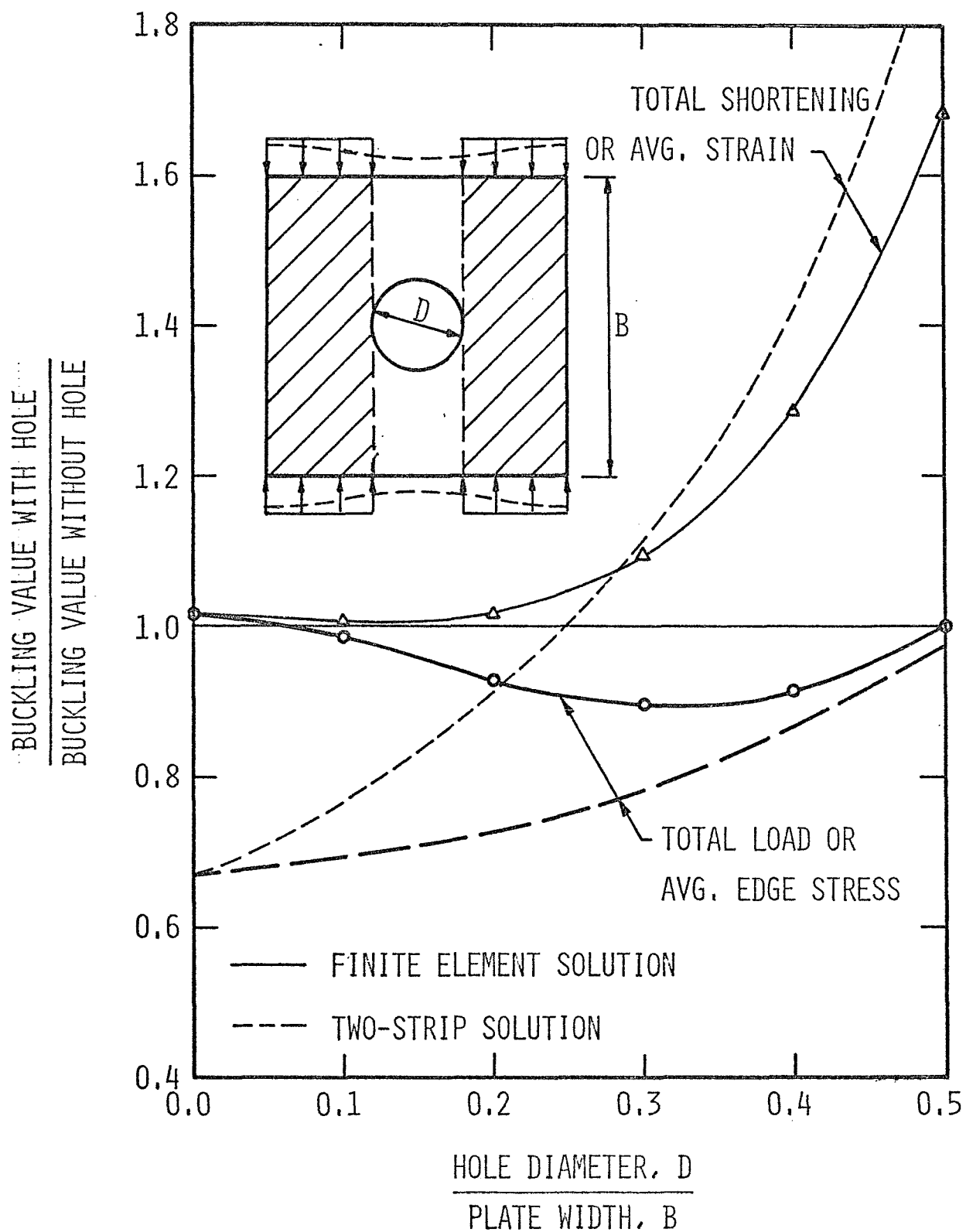


FIG. 5. EFFECT OF HOLE SIZE ON ELASTIC BUCKLING -- SIMPLY SUPPORTED SQUARE PLATE; UNREINFORCED HOLE; UNIFORM EDGE DISPLACEMENT

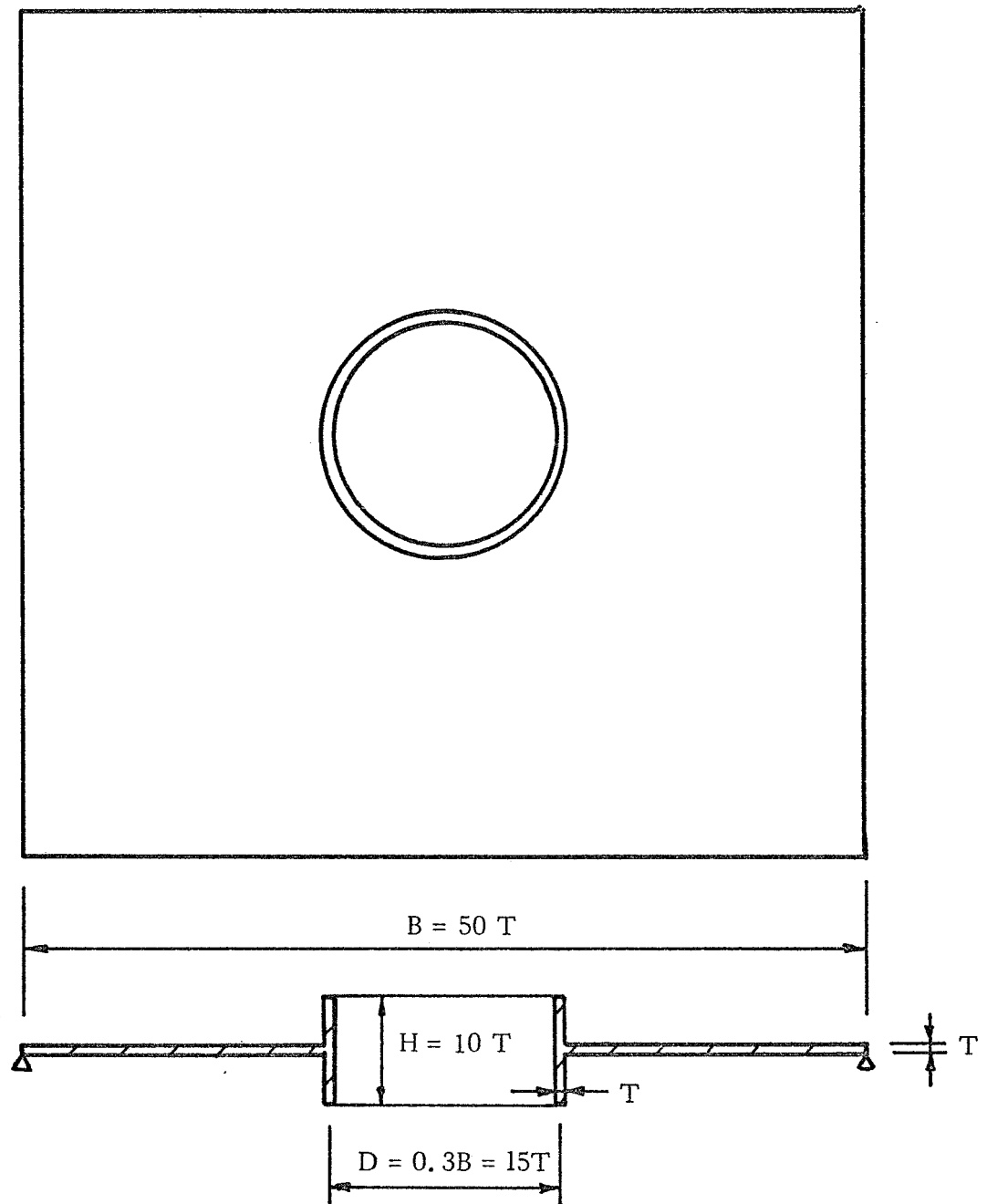


FIG. 6. PLATE WITH STIFFENED CIRCULAR HOLE

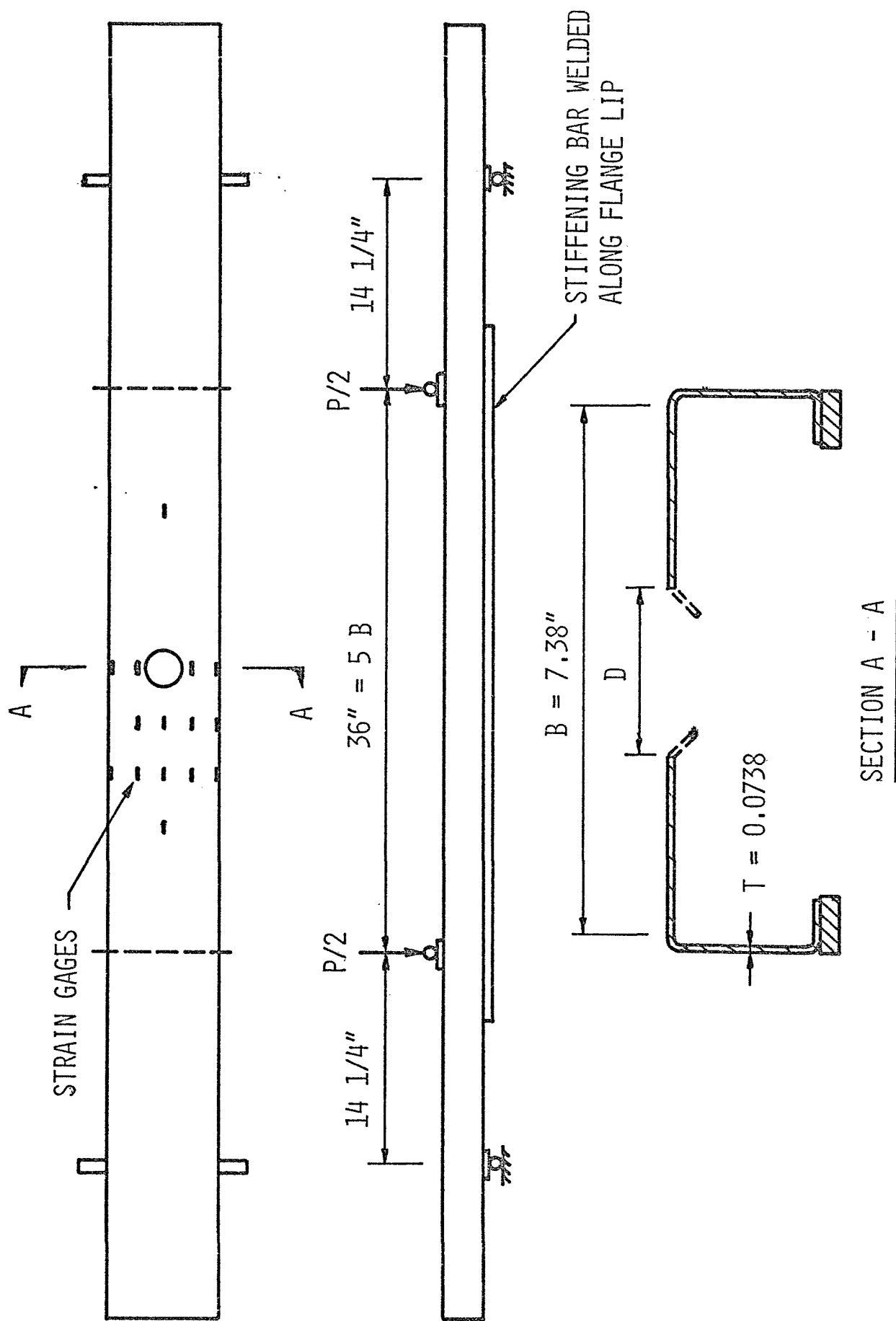


FIG. 7. SPECIMEN AND LOADING ARRANGEMENT FOR EXPERIMENTAL PROGRAM



FIG. 8. TEST SPECIMEN WITH ONE-INCH UNFLANGED HOLE

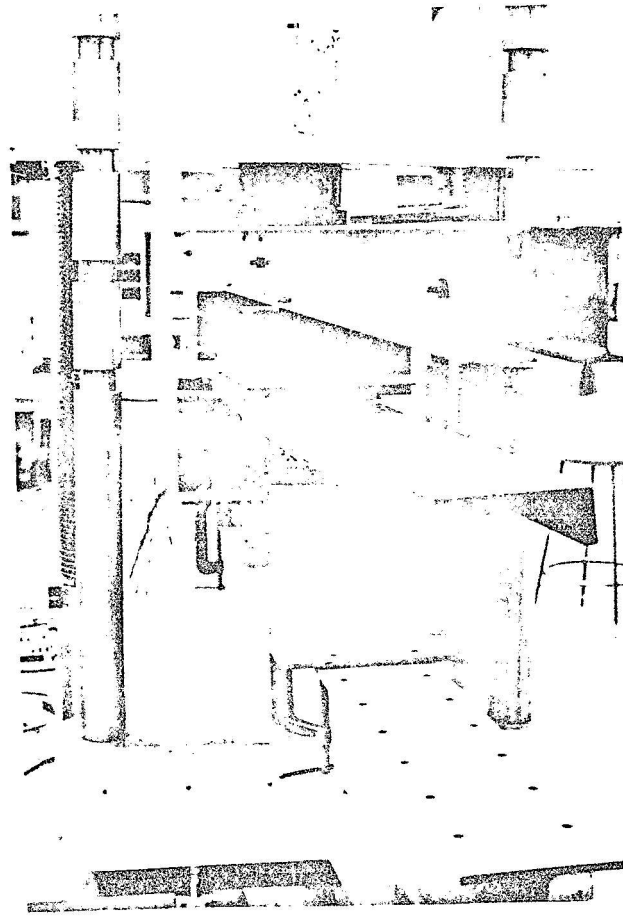


FIG. 9. TEST SET-UP IN UNIVERSAL TESTING MACHINE

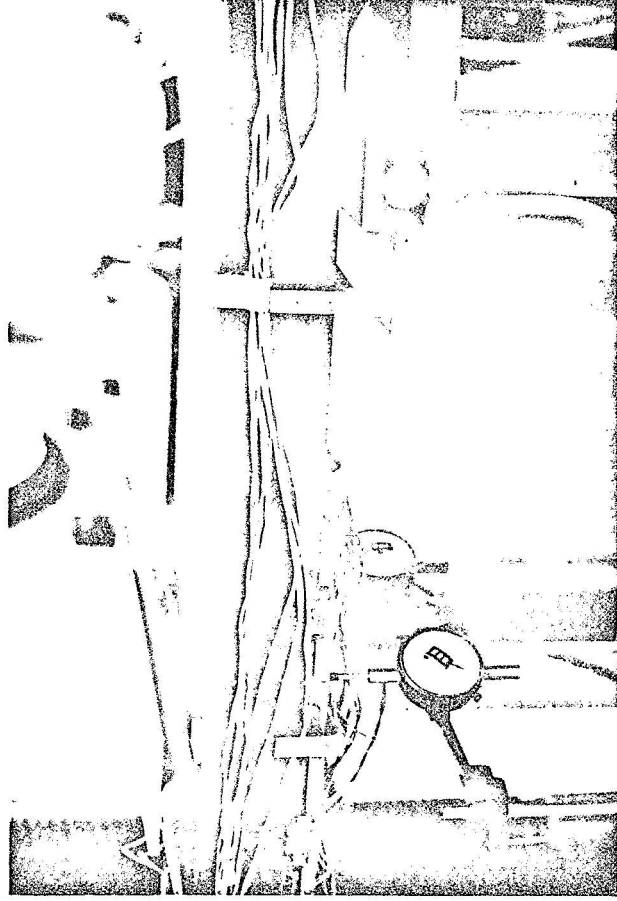


FIG. 10. SUPPORT AND LOAD POINT ARRANGEMENT AND FLANGE CLIPS

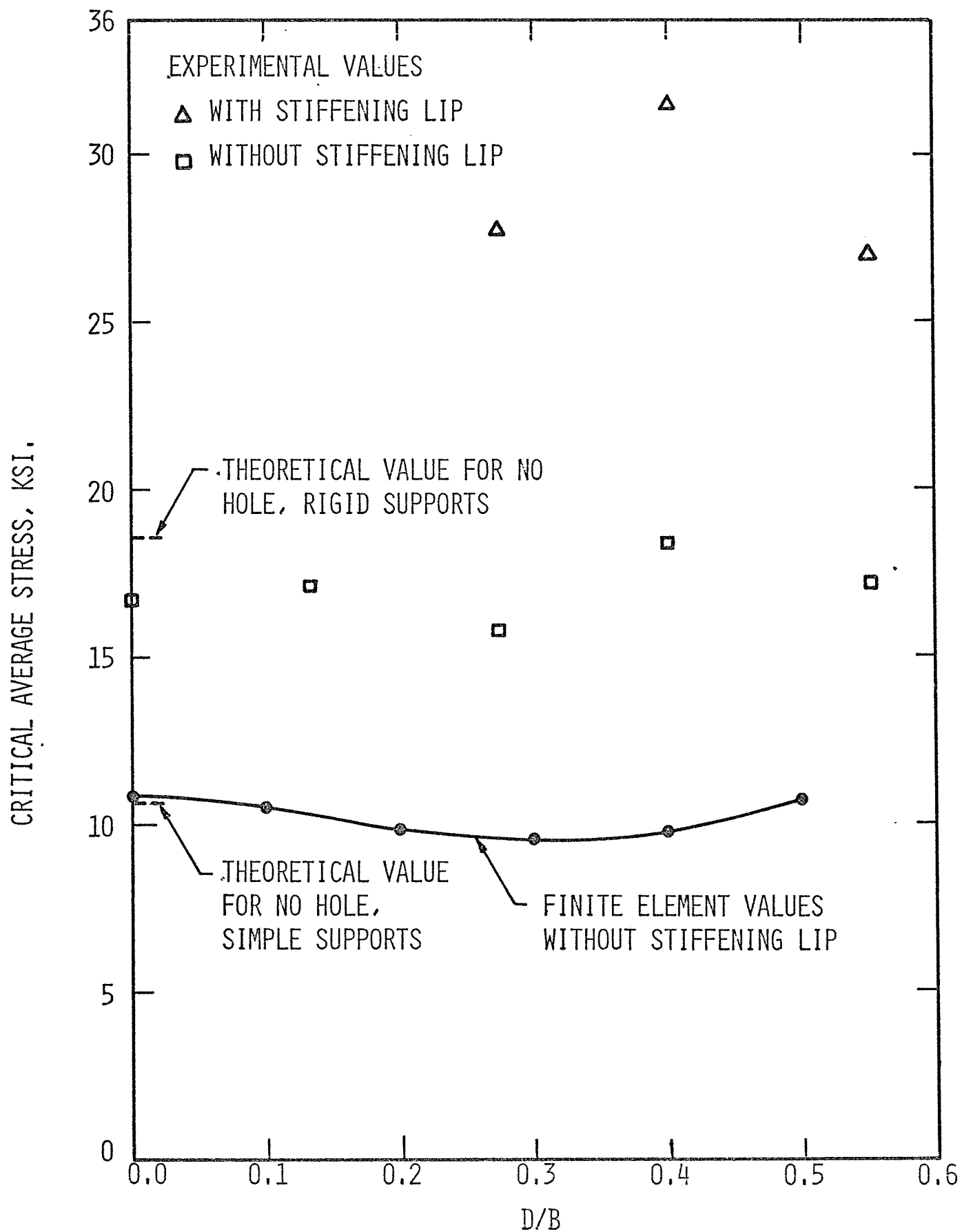


FIG. 11. EXPERIMENTAL RESULTS FOR ELASTIC BUCKLING

FIG. 12. PERFORATED WALL STUDS TESTED TO FAILURE

COMPRESSIVE BUCKLING OF PERFORATED PLATE ELEMENTS

By W. Pennington Vann

Errata in May, 1971 copies

1. Fig. 3, "E" should be changed to "e"
2. p. 3, paragraph 3, line 1: "sizes" should be changed to "size".
3. p. 5, paragraph 2, line 6: "... 3.615." should be changed to "... 3.615 with Poisson's ratio, $\nu = 0.3$."
4. p. 6, paragraph 3, line 8: "... resulting hole ..." should read "... resulting stiffened hole."
5. p. 10, paragraph 3, line 3: "... tend confirm..." should read "... tend to confirm..."
6. Fig. 11: the top ordinate should read "34," not "36"
7. Appendix -- Notation: add "T = plate thickness"